

Nom:

Prénom:

Factorisation ou décomposition en facteurs

Exercices supplémentaires

Factoriser les expressions suivantes **au maximum** :

a) $a^3 - 5a^2 + 5 - a$

b) $9a - a^3$

c) $n(n - m)^2 - (m - n)^3$

d) $n^2 - a^2 - 1 + 2a$

e) $x^3 + 9 - 9x - x^2$

f) $b^5 + 1 - b^3 - b^2$

g) $z^2 - 8z + 16$

h) $(a^2 - 1)^2 - (a + 2)^2$

i) $a^2 - b^2 + ax - bx$

j) $2y^2 - 18y + 28$

k) $-3x^4y^2 - 12x^3y + 3xy^2 + 3xy + 6x^2y^2$

l) $5cx + 10bx - 4cy^2 - 8by^2$

m) $(25x^2 + 30xy + 9y^2) - (36x^2 - 2xy + \frac{1}{36}y^2)$

n) $2(x - 2y) + (x - 2y)^2 - (x^2 - 4y^2) - (2x + 3y)(x - 2y)$

o) $(3x - 7)^2 - (2y + 5x)^2$

p) $1 - u^2 - x^2 - 2ux$

q) $ax - bx + 2x - ay + by - 2y$

r) $a^2 + ax - bx - b^2$

s) $z^2 - z - 12$

t) $x^3 + 5 - x^2 - 5x$

Solutions

- a) $a^3 - 5a^2 + 5 - a = a^2(a - 5) - (-5 + a) = (\mathbf{a - 5})(a^2 - 1)$
 $= (\mathbf{a - 5})(\mathbf{a + 1})(\mathbf{a - 1})$
- b) $9a - a^3 = a(9 - a^2) = \mathbf{a(3 + a)(3 - a)}$
- c) $n(n - m)^2 - (\mathbf{m - n})^3 = n(n - m)^2 - (-(\mathbf{n - m}))^3 = n(n - m)^2 + (n - m)^3$
 $= (n - m)^2(n + (n - m)) = (\mathbf{n - m})^2(\mathbf{2n - m})$
- d) $n^2 - a^2 - 1 + 2a = n^2 - a^2 + 2a - 1 = n^2 - (a^2 - 2a + 1) = n^2 - (a - 1)^2$
 $= (n + (a - 1))(n - (a - 1)) = (\mathbf{n + a - 1})(\mathbf{n - a + 1})$
- e) $x^3 + 9 - 9x - x^2 = x^3 - x^2 - 9x + 9 = x^2(x - 1) - 9(x - 1) = (x - 1)(x^2 - 9)$
 $= (\mathbf{x - 1})(\mathbf{x + 3})(\mathbf{x - 3})$
- f) $b^5 + 1 - b^3 - b^2 = b^5 - b^3 - b^2 + 1 = b^3(b^2 - 1) - (b^2 - 1) = (b^2 - 1)(b^3 - 1)$
 $= (b + 1)(b - 1)(b - 1)(b^2 + b + 1) = (\mathbf{b + 1})(\mathbf{b - 1})^2(\mathbf{b^2 + b + 1})$
- g) $z^2 - 8z + 16 = z^2 - 4z - 4z + 16 = z(z - 4) - 4(z - 4) = (z - 4)(z - 4)$
 $= (\mathbf{z - 4})^2$
- h) $(a^2 - 1)^2 - (a + 2)^2 = ((a^2 - 1) + (a + 2))((a^2 - 1) - (a + 2))$
 $= (a^2 - 1 + a + 2)(a^2 - 1 - a - 2) = (\mathbf{a^2 + a + 1})(\mathbf{a^2 - a - 3})$
- i) $a^2 - b^2 + ax - bx = (a^2 - b^2) + x(a - b) = (a + b)(a - b) + x(a - b)$
 $= (\mathbf{a - b})(\mathbf{a + b + x})$
- j) $2y^2 - 18y + 28 = 2(y^2 - 9y + 14) = 2(y^2 - 2y - 7y + 14)$
 $= 2(y(y - 2) - 7(y - 2)) = \mathbf{2(y - 2)(y - 7)}$
- k) $-3x^4y^2 - 12x^3y + 3xy^2 + 3xy + 6x^2y^2 = \mathbf{3xy(-x^3y - 4x^2 + y + 1 + 2xy)}$
- l) $5cx + 10bx - 4cy^2 - 8by^2 = 5x(c + 2b) - 4y^2(c + 2b) = (\mathbf{c + 2b})(\mathbf{5x - 4y^2})$
- m) $(25x^2 + 30xy + 9y^2) - (36x^2 - 2xy + \frac{1}{36}y^2) = (5x + 3y)^2 - (6x - \frac{1}{6}y)^2$
 $= ((5x + 3y) + (6x - \frac{1}{6}y))((5x + 3y) - (6x - \frac{1}{6}y))$
 $= (5x + 3y + 6x - \frac{1}{6}y)(5x + 3y - 6x + \frac{1}{6}y) = (11x + \frac{18}{6}y - \frac{1}{6}y)(-x + \frac{18}{6}y + \frac{1}{6}y)$
 $= (\mathbf{11x + \frac{17}{6}y})(\mathbf{-x + \frac{19}{6}})$
- n) $2(x - 2y) + (x - 2y)^2 - (x^2 - 4y^2) - (2x + 3y)(x - 2y)$
 $= 2(x - 2y) + (x - 2y)(x - 2y) - (x + 2y)(x - 2y) - (x - 2y)(2x + 3y)$
 $= (x - 2y)(2 + (x - 2y) - (x + 2y) - (2x + 3y))$
 $= (x - 2y)(2 + x - 2y - x - 2y - 2x - 3y) = (\mathbf{x - 2y})(\mathbf{-2x - 7y + 2})$
- o) $(3x - 7)^2 - (2y + 5x)^2 = ((3x - 7) + (2y + 5x))((3x - 7) - (2y + 5x))$
 $= (3x - 7 + 2y + 5x)(3x - 7 - 2y - 5x) = (\mathbf{8x + 2y - 7})(\mathbf{-2x - 2y - 7})$
- p) $1 - u^2 - x^2 - 2ux = 1 - u^2 - 2ux - x^2 = 1 - (u^2 + 2ux + x^2) = 1^2 - (u + x)^2$
 $= (1 + (u + x))(1 - (u + x)) = (\mathbf{1 + u + x})(\mathbf{1 - u - x})$
- q) $ax - bx + 2x - ay + by - 2y = x(a - b + 2) - y(a - b + 2) = (\mathbf{a - b + 2})(\mathbf{x - y})$
- r) $a^2 + ax - bx - b^2 = a^2 - b^2 + ax - bx = \mathbf{idem i)}$
- s) $z^2 - z - 12 = z^2 + 3z - 4z - 12 = z(z + 3) - 4(z + 3) = (\mathbf{z + 3})(\mathbf{z - 4})$
- t) $x^3 + 5 - x^2 - 5x = x^3 - x^2 - 5x + 5 = x^2(x - 1) - 5(x - 1) = (\mathbf{x - 1})(\mathbf{x^2 - 5})$