

Polynome faktorisieren

Übung 1

Faktorisieren Sie folgende Polynome so weit wie möglich durch einfaches ausklammern.

a) $16ax - 12ay - 8az$

$$= 4a(4x - 3y - 2z)$$

f) $33x^2y^2z + 66x^2yz^2 + 99xy^2z^2$

$$= 33xyz(xy + 2xz + 3yz)$$

b) $v^5 - v^3 + vw$

$$= v(v^4 - v^2 + w)$$

g) $9a^2b^2 - 12a^2b + 3ab$

$$= 3ab(3ab - 4a + 1)$$

c) $-p^3q^2 + p^5q^2 - p^3q^2r + 2p^3q^2$

$$\begin{aligned} &= p^3q^2(-1 + p^2 - r + 2) \\ &= p^3q^2(p^2 - r + 1) \end{aligned}$$

h) $m(a - b) + n(b - a)$

$$\begin{aligned} &= m(a - b) - n(a - b) \\ &= (a - b)(m - n) \end{aligned}$$

d) $49s + 51t - 28u$

$$= \text{unfaktorisierbar}$$

i) $n(x - y) - (x - y)$

$$= (x - y)(n - 1)$$

e) $2e^5 - 4e^3 + 8e$

$$= 2e(e^4 - 2e^2 + 4)$$

j) $(x + 1)(x - y) - (x - 3)(x - y) + (x - y)^2$

$$\begin{aligned} &= (x - y)((x + 1) - (x - 3) + (x - y)) \\ &= (x - y)(x + 1 - x + 3 + x - y) \\ &= (x - y)(x - y + 4) \end{aligned}$$

Übung 2

Faktorisieren Sie folgende Polynome so weit wie möglich durch doppeltes ausklammern.

a) $ac - ad + bc - bd$

$$\begin{aligned} &= a(c-d) + b(c-d) \\ &= (c-d)(a+b) \end{aligned}$$

b) $2x^3 - 3x^2 + 10x - 15$

$$\begin{aligned} &= x^2(2x-3) + 5(2x-3) \\ &= (2x-3)(x^2+5) \end{aligned}$$

c) $20ab - 1 + 4b - 5a$

$$\begin{aligned} &= 20ab - 5a + 4b - 1 \\ &= 5a(4b-1) + (4b-1) \\ &= (4b-1)(5a+1) \end{aligned}$$

d) $ab - ac - b + c$

$$\begin{aligned} &= a(b-c) - (b-c) \\ &= (b-c)(a-1) \end{aligned}$$

e) $-6a^4 + 10a^3 + 15a - 25$

$$\begin{aligned} &= -2a^3(3a-5) + 5(2a-5) \\ &= (3a-5)(5-2a^3) \end{aligned}$$

f) $25xy^3 - 50y^3z - 15x^2y^2 + 30xy^2z$

$$\begin{aligned} &= 5y^2(5xy - 10yz - 3x^2 + 6xz) \\ &= 5y^2(5y(x-2z) - 3x(x-2z)) \\ &= 5y^2(x-2z)(5y-3x) \end{aligned}$$

g) $10abc - 4 - 2a + 20bc$

$$\begin{aligned} &= 10abc + 20bc - 2a - 4 \\ &= 10bc(a+2) - 2(a+2) \\ &= (a+2)(10bc-2) \\ &= 2(a+2)(5bc-1) \end{aligned}$$

h) $ax - bx + 2x - ay + by - 2y$

$$\begin{aligned} &= x(a-b+2) - y(a-b+2) \\ &= (a-b+2)(x-y) \end{aligned}$$

i) $30r^2 + 30s - 45rs - 20t - 20r + 30rt$

$$\begin{aligned} &= 30r^2 - 20r + 30s - 45rs + 30rt - 20t \\ &= 10r(3r-2) - 15s(3r-2) + 10t(3r-2) \\ &= (3r-2)(10r - 15s + 10t) \\ &= 5(3r-2)(2r - 3s + 2t) \end{aligned}$$

j) $2ax - b - 5ay - 2bx + a + 5by$

$$\begin{aligned} &= 2ax - 2bx - 5ay + 5by + a - b \\ &= 2x(a-b) - 5y(a-b) + (a-b) \\ &= (a-b)(2x - 5y + 1) \end{aligned}$$

Übung 3

Faktorisieren Sie folgende Polynome so weit wie möglich anhand der zwei ersten binomischen Formeln.

a) $p^2 + 2pq + q^2$

$$= (p + q)^2$$

f) $x^4y^2 + 2x^2y + 1$

$$= (x^2y + 1)^2$$

b) $4e^2 - 4e + 1$

$$= (2e - 1)^2$$

g) $2m^4 - 16m^2 + 32$

$$= 2(m^4 - 8m^2 + 16)$$

$$= 2(m^2 - 4)^2$$

$$= 2(m + 2)^2(m - 2)^2$$

c) $25a^6 - 20a^3b + 4b^2$

$$= (5a^3 - 2b)^2$$

h) $-r^4 + 2r^3s - r^2s^2$

$$= -r^2(r^2 - 2rs + s^2)$$

$$= -r^2(r - s)^2$$

d) $-24x^2 - 24x - 6$

$$= -6(4x^2 + 4x + 1)$$

$$= -6(2x + 1)^2$$

i) $25x^2 - 10x + 1$

$$= (5x - 1)^2$$

e) $x^2 - 3xy + y^2$

$$= \text{unfaktorierbar}$$

j) $\frac{1}{9}a^4 - \frac{5}{6}a^2b^2 + \frac{25}{16}b^4$

$$= \left(\frac{1}{3}a^2 - \frac{5}{4}b^2\right)^2$$

Übung 4

Faktorisieren Sie folgende Polynome so weit wie möglich anhand der 3ten binomischen Formel.

a) $1 - x^2$

$$= (1+x)(1-x)$$

f) $4(a-b)^2 - 81c^2$

$$\begin{aligned} &= (2(a-b))^2 - (9c)^2 \\ &= (2a-2b+9c)(2a-2b-9c) \end{aligned}$$

b) $u^3 - 25u$

$$\begin{aligned} &= u(u^2 - 25) \\ &= u(u+5)(u-5) \end{aligned}$$

g) $h^4 - 16$

$$\begin{aligned} &= (h^2+4)(h^2-4) \\ &= (h^2+4)(h+2)(h-2) \end{aligned}$$

c) $2x^3y - 18xy^3$

$$\begin{aligned} &= 2xy(x^2 - 9y^2) \\ &= 2xy(x+3y)(x-3y) \end{aligned}$$

h) $(a^2 - 2)^2 - (a+1)^2$

$$\begin{aligned} &= (a^2 - 2 + a + 1)(a^2 - 2 - a - 1) \\ &= (a^2 + a - 1)(a^2 - a - 3) \end{aligned}$$

d) $9a^2 - 49b^2$

$$= (3a+7b)(3a-7b)$$

i) $(2d-1)^2 - 36(2-d)^2$

$$\begin{aligned} &= (2d-1)^2 - (6(2-d))^2 \\ &= (2d-1+12-6d)(2d-1-12+6d) \\ &= (-4d+11)(8d-13) \end{aligned}$$

e) $25x^2 - (2x+3)^2$

$$\begin{aligned} &= (5x)^2 - (2x+3)^2 \\ &= (5x+2x+3)(5x-2x-3) \\ &= (7x+3)(3x-3) \\ &= 3(7x+3)(x-1) \end{aligned}$$

j) $100t^2 - 9(u-t)^2$

$$\begin{aligned} &= (10t)^2 - (3(u-t))^2 \\ &= (10t+3u-3t)(10t-3u+3t) \\ &= (7t+3u)(13t-3u) \end{aligned}$$

Übung 5

Faktorisieren Sie folgende Polynome indem Sie diese anhand der ersten und zweiten binomischen Formel in die Form $a^2 - b^2 = (a + b)(a - b)$ bringen.

a) $a^2 + 2ab + b^2 - 36z^2$

$$= (a+b)^2 - (6z)^2$$

$$= (a+b+6z)(a+b-6z)$$

f) $4x^2 - 12x + 9 - (a+3)^2$

$$= (2x-3)^2 - (a+3)^2$$

$$= (2x-3+a+3)(2x-3-a-3)$$

$$= (2x+a)(2x-a-6)$$

b) $p^2 - x^2 - 2x - 1$

$$= p^2 - (x^2 + 2x + 1)$$

$$= p^2 - (x+1)^2$$

$$= (p+x+1)(p-x-1)$$

g) $9x^2 - y^2 - 2yz - z^2$

$$= 9x^2 - (y^2 + 2yz + z^2)$$

$$= (3x)^2 - (y+z)^2$$

$$= (3x+y+z)(3x-y-z)$$

c) $u^2 - 8uv + 16v^2 - 1$

$$= (u-4v)^2 - 1$$

$$= (u-4v+1)(u-4v-1)$$

h) $36(a-b)^2 - x^2 + 6x - 9$

$$= (6(a-b))^2 - (x^2 - 6x + 9)$$

$$= (6a-6b)^2 - (x-3)^2$$

$$= (6a-6b+x-3)(6a-6b-x+3)$$

d) $m^2 - q^2 + 10q - 25$

$$= m^2 - (q^2 - 10q + 25)$$

$$= m^2 - (q-5)^2$$

$$= (m+q-5)(m-q+5)$$

i) $16p^2q^2 - p^2 - 2pq - q^2$

$$= (4pq)^2 - (p^2 + 2pq + q^2)$$

$$= (4pq)^2 - (p+q)^2$$

$$= (4pq+p+q)(4pq-p-q)$$

e) $x^2 - 2xy + y^2 - a^2 - 2ab - b^2$

$$= (x-y)^2 - (a^2 + 2ab + b^2)$$

$$= (x-y)^2 - (a+b)^2$$

$$= (x-y+a+b)(x-y-a-b)$$

j) $a^2 - c^2 - 2ab + b^2$

$$= a^2 - 2ab + b^2 - c^2$$

$$= (a-b)^2 - c^2$$

$$= (a-b+c)(a-b-c)$$

Übung 6

Faktorisieren Sie folgende Polynome anhand des Zweikammeransatzes.

a) $a^2 + 12a + 20$

$$= (a+2)(a+10)$$

f) $x^4 - 7x^2 + 12$

$$= (x^2 - 3)(x^2 - 4)$$

$$= (x^2 - 3)(x+2)(x-2)$$

b) $b^2 - 12b + 20$

$$= (b-2)(b-10)$$

g) $t^2 + 9t + 20$

$$= (t+5)(t+4)$$

c) $x^2 - 3x - 10$

$$= (x-5)(x+2)$$

h) $e^2 + e - 2$

$$= (e+2)(e-1)$$

d) $e^2 - e - 2$

$$= (e-2)(e+1)$$

i) $x^2 - 15x + 56$

$$= (x-8)(x-7)$$

e) $2a^2b + 2ab - 40b$

$$= 2b(a^2 + a - 20)$$

$$= 2b(a+5)(a-4)$$

j) $y^2 - 51y + 144$

$$= (y-48)(y-3)$$

Übung 7

Faktorisieren Sie folgende Polynome anhand der Zerlegung eines quadratischen Trinoms.

a) $4k^2 - 5k + 1$

$$\begin{aligned} &= 4k^2 - 4k - k + 1 \\ &= 4k(k-1) - (k-1) \\ &= (k-1)(4k-1) \end{aligned}$$

f) $12r^2 - 8r + 1$

$$\begin{aligned} &= 12r^2 - 2r - 6r + 1 \\ &= 2r(6r-1) - (6r-1) \\ &= (6r-1)(2r-1) \end{aligned}$$

b) $2x^2 + 11x + 5$

$$\begin{aligned} &= 2x^2 + 10x + x + 5 \\ &= 2x(x+5) + (x+5) \\ &= (x+5)(2x+1) \end{aligned}$$

g) $2b^5 + 9b^4 - 5b^3$

$$\begin{aligned} &= b^3(2b^2 + 9b - 5) \\ &= b^3(2b^2 + 10b - b - 5) \\ &= b^3(2b(b+5) - (b+5)) \\ &= b^3(b+5)(2b-1) \end{aligned}$$

c) $6z^2 - z - 1$

$$\begin{aligned} &= 6z^2 - 3z + 2z - 1 \\ &= 3z(2z-1) + (2z-1) \\ &= (2z-1)(3z+1) \end{aligned}$$

h) $4x^4 + 3x^2 - 1$

$$\begin{aligned} &= 4x^4 + 4x^2 - x^2 - 1 \\ &= 4x^2(x^2+1) - (x^2+1) \\ &= (x^2+1)(4x^2-1) \\ &= (x^2+1)(2x+1)(2x-1) \end{aligned}$$

d) $-2r^2 + 5r - 2$

$$\begin{aligned} &= -2r^2 + 4r + r - 2 \\ &= -2r(r-2) + (r-2) \\ &= (r-2)(1-2r) \end{aligned}$$

i) $3c^2 + 16c + 5$

$$\begin{aligned} &= 3c^2 + 15c + c + 5 \\ &= 3c(c+5) + (c+5) \\ &= (c+5)(3c+1) \end{aligned}$$

e) $5z^2 - 2z - 3$

$$\begin{aligned} &= 5z^2 - 5z + 3z - 3 \\ &= 5z(z-1) + 3(z-1) \\ &= (z-1)(5z+3) \end{aligned}$$

j) $6x^2 + x - 15$

$$\begin{aligned} &= 6x^2 + 10x - 9x - 15 \\ &= 2x(3x+5) - 3(3x+5) \\ &= (3x+5)(2x-3) \end{aligned}$$

