

1 : Fractions rationnelles

lundi, 25 novembre 2024

15:49



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Simplification de fractions rationnelles

Exercice 1

Simplifier au maximum les fractions rationnelles suivantes :

a) $\frac{x^2 - 25}{x^2 + 5x} =$

$$= \frac{\cancel{(x+5)}(x-5)}{x\cancel{(x+5)}}$$

$$= \boxed{\frac{x-5}{x}}$$

d) $\frac{-x^2 + x + 12}{x^3 + 3x^2} =$

$$= \frac{-(x-4)\cancel{(x+3)}}{x^2\cancel{(x+3)}}$$

$$= \boxed{\frac{-(x-4)}{x^2}}$$

b) $\frac{2x^2 + 7x + 3}{2x^2 - 7x - 4} =$

$$= \frac{\cancel{(2x+1)}(x+3)}{\cancel{(2x+1)}(x-4)}$$

$$= \boxed{\frac{x+3}{x-4}}$$

e) $\frac{18x + 18}{12x^2 - 12} =$

$$= \frac{\overset{3}{\cancel{18}}(x+1)}{\underset{2}{\cancel{12}}(x^2-1)} = \frac{3\cancel{(x+1)}}{2\cancel{(x+1)}(x-1)}$$

$$= \boxed{\frac{3}{2(x-1)}}$$

c) $\frac{14x^2 - 7x}{8x^2 - 8x + 2} =$

$$= \frac{7x(2x-1)}{2(4x^2-4x+1)} = \frac{7x\cancel{(2x-1)}}{2(2x-1)^2}$$

$$= \boxed{\frac{7x}{2(2x-1)}}$$

f) $\frac{3x^3 + 6x^2 + 3x}{3x^3 + 9x^2 + 6x} =$

$$= \frac{\cancel{3x}(x^2+2x+1)}{\cancel{3x}(x^2+3x+2)}$$

$$= \frac{(x+1)^2}{\cancel{(x+1)}(x+2)} = \boxed{\frac{x+1}{x+2}}$$

Exercice 2

Simplifier au maximum les multiplications de fractions rationnelles suivantes :

$$\begin{aligned}
 \text{a) } & \frac{9x^2 - 4}{9x^2 + 6x} \cdot \frac{6x^3}{3x^2 - 5x + 2} = \\
 & = \frac{\cancel{(3x+2)} \cancel{(3x-2)} \cdot 6x^3}{3x \cancel{(3x+2)} \cdot \cancel{(3x-2)} (x-1)} = \frac{\overset{2}{\cancel{6}} \overset{3}{\cancel{x^3}}}{\cancel{3} (x-1)} \\
 & = \boxed{\frac{2x^2}{x-1}}
 \end{aligned}$$

$$\begin{aligned}
 \text{b) } & \frac{8x^2y}{x^2 + 2xy + y^2} \cdot \frac{x^2 - y^2}{6x^2y^2 - 6xy^3} = \\
 & = \frac{8x^2y \cdot \cancel{(x+y)} \cancel{(x-y)}}{(\cancel{x+y})^2 \cdot 6xy^2 \cancel{(x-y)}} = \frac{\overset{4}{\cancel{8}} x^2 \cancel{y}}{(\cancel{x+y}) \overset{3}{\cancel{6}} xy^2} \\
 & = \boxed{\frac{4x}{3y(x+y)}}
 \end{aligned}$$

$$\begin{aligned}
 \text{c) } & \frac{x^2 - 3x - 10}{x^2 + 2x - 35} \cdot \frac{x^2 + 4x - 21}{x^2 + 9x + 14} = \\
 & = \frac{\cancel{(x+2)} \cancel{(x-5)} \cdot \cancel{(x+7)} (x-3)}{(x+7) \cancel{(x-5)} \cdot \cancel{(x+2)} \cancel{(x+7)}} \\
 & = \boxed{\frac{x-3}{x+7}}
 \end{aligned}$$

$$\begin{aligned}
 \text{d) } & \frac{16 - x^2}{-4x^2 + 20x - 16} \cdot \frac{2x^2 - 4x + 2}{x^2 + 3x - 4} = \\
 & = \frac{(4-x)(4+x) \cdot 2(x^2 - 2x + 1)}{-4(x^2 - 5x + 4) \cdot (x+4)(x-1)} = \frac{(4-x) \cancel{(4+x)} \cdot 2 \cdot \cancel{(x-1)}^2}{-4 \cancel{(x-1)} (x-4) \cdot \cancel{(x+4)} \cancel{(x-1)}} \\
 & = \frac{(4-x) \cdot 2}{-4(x-4)} = \frac{\overset{2}{\cancel{2}} \cancel{(4-x)}}{\underset{2}{\cancel{4}} (4-x)} = \boxed{\frac{1}{2}}
 \end{aligned}$$

Exercice 3

Simplifier au maximum les divisions de fractions rationnelles suivantes :

$$\text{a) } \frac{x^2 - 9}{3x^2 - 6} \div \frac{x^2 + 3x}{3x^4 - 12} =$$

$$= \frac{(x+3)(x-3)}{3(x^2-2)} \cdot \frac{3(x^4-4)}{x(x+3)} = \frac{\cancel{(x+3)}(x-3) \cdot \cancel{3}(x^2+2)\cancel{(x^2-2)}}{\cancel{3}(x^2-2) \cdot x \cancel{(x+3)}}$$

$$= \boxed{\frac{(x-3)(x^2+2)}{x}}$$

$$\text{b) } \frac{5a^2 + 12a + 4}{a^4 - 16} \div \frac{25a^2 + 20a + 4}{a^2 - 2a} =$$

$$= \frac{(5a+2)(a+2)}{(a^2-4)(a^2+4)} \cdot \frac{a(a-2)}{(5a+2)^2} = \frac{\cancel{(5a+2)}\cancel{(a+2)} \cdot a\cancel{(a-2)}}{\cancel{(a+2)}\cancel{(a-2)}(a^2+4) \cdot \cancel{(5a+2)}^2}$$

$$= \boxed{\frac{a}{(a^2+4)(5a+2)}}$$

$$\text{c) } \frac{8x^2 - 9x + 1}{x^2 - 1} \div \frac{x^2 - 6x}{x^2 - 5x - 6} =$$

$$= \frac{(8x-1)\cancel{(x-1)}}{\cancel{(x+1)}\cancel{(x-1)}} \cdot \frac{\cancel{(x-6)}\cancel{(x+1)}}{x\cancel{(x-6)}}$$

$$= \boxed{\frac{8x-1}{x}}$$

$$\text{d) } \frac{x^2 - x - 6}{x^2 - 4x - 5} \div \frac{x^2 + 2x - 15}{x^2 - 25} =$$

$$= \frac{(\underline{x+2})\cancel{(x-3)}}{(x+1)\cancel{(x-5)}} \cdot \frac{\cancel{(x+5)}\cancel{(x-5)}}{\cancel{(x+5)}\cancel{(x-3)}}$$

$$= \boxed{\frac{x+2}{x+1}}$$

Exercice 4

Simplifier au maximum les additions et soustractions de fractions rationnelles suivantes :

$$\text{a) } \frac{2x-1}{3} + \frac{x-2}{15} + \frac{3x+1}{5}$$

$$= \frac{5(2x-1)}{15} + \frac{x-2}{15} + \frac{3(3x+1)}{15} = \frac{(10x-5)+(x-2)+(9x+3)}{15}$$

$$= \boxed{\frac{20x-4}{15}}$$

$$\text{b) } \frac{x-4}{3} + \frac{x-3}{4} - \frac{x-12}{12} =$$

$$= \frac{4(x-4)}{12} + \frac{3(x-3)}{12} - \frac{(x-12)}{12} = \frac{4x-16+3x-9-x+12}{12}$$

$$= \boxed{\frac{6x-13}{12}}$$

$$\text{c) } \frac{5}{x} - \frac{2x-1}{x^2} + \frac{x+5}{x^3} =$$

$$= \frac{5x^2}{x^3} - \frac{x(2x-1)}{x^3} + \frac{(x+5)}{x^3} = \frac{5x^2-2x^2+x+x+5}{x^3}$$

$$= \boxed{\frac{3x^2+2x+5}{x^3}}$$

$$\text{d) } \frac{2}{x-2} - \frac{3}{x+1}$$

$$= \frac{2(x+1)}{(x-2)(x+1)} - \frac{3(x-2)}{(x-2)(x+1)} = \frac{2x+2-3x+6}{(x-2)(x+1)}$$

$$= \boxed{\frac{-x+8}{(x-2)(x+1)}}$$

$$\begin{aligned}
 \text{e) } \frac{x^2 + 5x - 4}{x^2 - 16} - \frac{1}{x + 4} &= \frac{x^2 + 5x - 4}{(x+4)(x-4)} - \frac{1 \cdot (x-4)}{(x+4)(x-4)} \\
 &= \frac{x^2 + 5x - 4 - x + 4}{(x+4)(x-4)} = \frac{x^2 + 4x}{(x+4)(x-4)} = \frac{x \cancel{(x+4)}}{\cancel{(x+4)}(x-4)} \\
 &= \boxed{\frac{x}{x-4}}
 \end{aligned}$$

$$\begin{aligned}
 \text{f) } \frac{x}{x^2 - 7x + 6} - \frac{2x}{x^2 - 2x - 24} &= \frac{x}{(x-1)(x-6)} - \frac{2x}{(x-6)(x+4)} \\
 &= \frac{x(x+4) - 2x(x-1)}{(x-1)(x-6)(x+4)} = \frac{x^2 + 4x - 2x^2 + 2x}{(x-1)(x-6)(x+4)} \\
 &= \frac{-x^2 + 6x}{(x-1)(x-6)(x+4)} = \frac{-x \cancel{(x-6)}}{(x-1)\cancel{(x-6)}(x+4)} = \boxed{\frac{-x}{(x-1)(x+4)}}
 \end{aligned}$$

$$\begin{aligned}
 \text{g) } \frac{2}{x^2 + 3x} + \frac{12}{x^3 - 9x} &= \frac{2}{x(x+3)} + \frac{12}{x(x+3)(x-3)} \\
 &= \frac{2(x-3) + 12}{x(x+3)(x-3)} = \frac{2x + 6}{x(x+3)(x-3)} = \frac{2 \cancel{(x+3)}}{x \cancel{(x+3)}(x-3)} \\
 &= \boxed{\frac{2}{x(x-3)}}
 \end{aligned}$$

$$\begin{aligned}
 \text{h) } \frac{4}{x-1} - \frac{2}{x+2} &= \frac{4(x+2)}{(x-1)(x+2)} - \frac{2(x-1)}{(x-1)(x+2)} = \frac{4x + 8 - 2x + 2}{(x-1)(x+2)} \\
 &= \boxed{\frac{2x + 10}{(x-1)(x+2)}}
 \end{aligned}$$

$$\begin{aligned}
 \text{i) } \frac{5}{6x^3 - 6x^2} + \frac{14}{x^2 - 1} &= \frac{5}{6x^2(x-1)} + \frac{14}{(x+1)(x-1)} \\
 &= \frac{5(x+1) + 14 \cdot 6x^2}{6x^2(x+1)(x-1)} = \boxed{\frac{84x^2 + 5x + 5}{6x^2(x+1)(x-1)}}
 \end{aligned}$$

$$\begin{aligned}
 \text{j) } \frac{3x}{x+2} + \frac{5x}{x-2} - \frac{40}{x^2-4} &= \frac{3x(x-2)}{(x+2)(x-2)} + \frac{5x(x+2)}{(x+2)(x-2)} - \frac{40}{(x+2)(x-2)} \\
 &= \frac{3x^2 - 6x + 5x^2 + 10x - 40}{(x+2)(x-2)} = \frac{8x^2 + 4x - 40}{(x+2)(x-2)} = \frac{4(2x^2 + x - 10)}{(x+2)(x-2)} \\
 &= \frac{4(2x+5)\cancel{(x-2)}}{(x+2)\cancel{(x-2)}} = \boxed{\frac{4(2x+5)}{x+2}}
 \end{aligned}$$

$$\begin{aligned}
 \text{k) } 2 - \frac{5}{x} - \frac{2x}{x+5} &= \frac{2x(x+5)}{x(x+5)} - \frac{5(x+5)}{x(x+5)} - \frac{2x \cdot x}{x(x+5)} \\
 &= \frac{2x^2 + 10x - 5x - 25 - 2x^2}{x(x+5)} = \boxed{\frac{5x - 25}{x(x+5)}}
 \end{aligned}$$

$$\begin{aligned}
 \text{l) } \frac{x^2 - 6x + 8}{x^2 + 3x + 2} - \frac{2}{x+2} &= \frac{x^2 - 6x + 8}{(x+2)(x+1)} - \frac{2(x+1)}{(x+2)(x+1)} \\
 &= \frac{x^2 - 6x + 8 - 2x - 2}{(x+2)(x+1)} = \boxed{\frac{x^2 - 8x + 6}{(x+2)(x+1)}}
 \end{aligned}$$

$$\begin{aligned}
 \text{m)} \quad & \frac{3x}{x^2 - 8x + 7} - \frac{x}{x^2 - 2x - 35} = \frac{3x}{(x-1)(x-7)} - \frac{x}{(x+5)(x-7)} \\
 & = \frac{3x(x+5) - x(x-1)}{(x-1)(x-7)(x+5)} = \frac{3x^2 + 15x - x^2 + x}{(x-1)(x-7)(x+5)} \\
 & = \boxed{\frac{2x^2 + 16x}{(x-1)(x-7)(x+5)}}
 \end{aligned}$$

$$\begin{aligned}
 \text{n)} \quad & \frac{x^2 - 1}{x + 2} + \frac{x^2}{2 - x} + \frac{5x^2 - 3x + 2}{x^2 - 4} = \frac{(x^2 - 1)(x-2)}{(x+2)(x-2)} - \frac{x^2(x+2)}{(x-2)(x+2)} + \frac{5x^2 - 3x + 2}{(x-2)(x+2)} \\
 & = \frac{x^3 - x - 2x^2 + 2 - x^3 - 2x^2 + 5x^2 - 3x + 2}{(x+2)(x-2)} = \frac{x^2 - 4x + 4}{(x+2)(x-2)} \\
 & = \frac{(x-2)^2}{(x+2)(x-2)} = \boxed{\frac{x-2}{x+2}}
 \end{aligned}$$

$$\begin{aligned}
 \text{o)} \quad & \frac{4x}{3x-4} + \frac{8}{3x^2-4x} + \frac{2}{x} = \frac{4x \cdot x}{(3x-4) \cdot x} + \frac{8}{x(3x-4)} + \frac{2 \cdot (3x-4)}{x \cdot (3x-4)} \\
 & = \frac{4x^2 + 8 + 6x - 8}{(3x-4) \cdot x} = \frac{4x^2 + 6x}{(3x-4) \cdot x} = \frac{x(4x+6)}{(3x-4) \cdot x} = \boxed{\frac{4x+6}{3x-4}}
 \end{aligned}$$

$$\begin{aligned}
 \text{p)} \quad & \frac{2x+1}{x^2+4x+4} - \frac{6x}{x^2-4} + \frac{3}{x-2} = \frac{2x+1}{(x+2)^2} - \frac{6x}{(x+2)(x-2)} + \frac{3}{x-2} \\
 & = \frac{(2x+1)(x-2)}{(x+2)^2(x-2)} - \frac{6x(x+2)}{(x+2)^2(x-2)} + \frac{3(x+2)^2}{(x+2)^2(x-2)} \\
 & = \frac{2x^2 - 4x + x - 2 - 6x^2 - 12x + 3x^2 + 12x + 12}{(x+2)^2(x-2)} = \frac{-x^2 - 3x + 10}{(x+2)^2(x-2)} \\
 & = \frac{-(x-2)(x+5)}{(x+2)^2(x-2)} = \boxed{\frac{-(x+5)}{(x+2)^2}}
 \end{aligned}$$