

1 : Equations rationnelles

lundi, 23 février 2026 12:05

Nom:

Prénom:

Date:



ÉCOLE DES MÉTIERS FRIBOURG
BERUFSFACHSCHULE FREIBURG
MATURITÉ | MATURITÄT

Equations rationnelles

Donner le domaine de définition $\mathbb{D}$, résoudre, puis donner l'ensemble de solutions $S = \{\dots\}$ des équations ci-dessous.

1) $\frac{4x+6}{1-x} = 8$ $\left| \cdot (1-x) \right.$

$\mathbb{D} = \mathbb{R} \setminus \{1\}$

$4x+6 = 8(1-x)$

$4x+6 = 8-8x$ $\left| +8x-6 \right.$

$12x = 2$ $\left| \div 12 \right.$

$x = \frac{2}{12} = \frac{1}{6} \in \mathbb{D} \checkmark$

$S = \left\{ \frac{1}{6} \right\}$

2) $\frac{x-1}{x+3} + 5 = 0$ $\left| \cdot (x+3) \right.$

$\mathbb{D} = \mathbb{R} \setminus \{-3\}$

$x-1+5(x+3)=0$

$x-1+5x+15=0$ $\left| -14 \right.$
 $6x = -14$ $\left| \div 6 \right.$

$x = \frac{-14}{6} = -\frac{7}{3} \in \mathbb{D} \checkmark$

$S = \left\{ -\frac{7}{3} \right\}$

3) $\frac{3}{x+4} = \frac{-5}{x-2}$ $\left| \cdot (x+4)(x-2) \right.$

$\mathbb{D} = \mathbb{R} \setminus \{-4; 2\}$

$3(x-2) = -5(x+4)$

$3x-6 = -5x-20$ $\left| +5x+6 \right.$
 $8x = -14$ $\left| \div 8 \right.$

$x = \frac{-14}{8} = -\frac{7}{4} \in \mathbb{D} \checkmark$

$S = \left\{ -\frac{7}{4} \right\}$

4) $2x = \frac{x+1}{2x-1}$ $\left| \cdot (2x-1) \right.$

$\mathbb{D} = \mathbb{R} \setminus \left\{ \frac{1}{2} \right\}$

$2x(2x-1) = x+1$

$4x^2-2x = x+1$ $\left| -x-1 \right.$

$4x^2-3x-1=0$

$\Delta = (-3)^2 - 4 \cdot 4 \cdot (-1) = 9+16 = 25$

$x_{1,2} = \frac{3 \pm 5}{8}$ $\begin{cases} \frac{8}{8} = 1 \in \mathbb{D} \checkmark \\ \frac{-2}{8} = -\frac{1}{4} \in \mathbb{D} \checkmark \end{cases}$

$S = \left\{ 1; -\frac{1}{4} \right\}$

$$5) \frac{6}{x-5} + \frac{x}{5-x} = 1$$

$$D = \mathbb{R} \setminus \{5\}$$

$$\frac{6}{x-5} - \frac{x}{x-5} = 1 \quad | \cdot (x-5)$$

$$6 - x = x - 5 \quad | +x+5$$

$$11 = 2x$$

$$\frac{11}{2} = x \in D \checkmark$$

$$S = \left\{ \frac{11}{2} \right\}$$

$$6) \frac{4}{x} + \frac{x}{x+1} = \frac{x^2}{x^2+x} \quad | \cdot x(x+1)$$

$$D = \mathbb{R} \setminus \{0; -1\}$$

$$4(x+1) + x^2 = x^2 \quad | -x^2$$

$$4x + 4 = 0 \quad | +4$$

$$4x = -4 \quad | :4$$

$$x = -1 \notin D !$$

$$S = \emptyset$$

$$7) \frac{7+x}{7-x} - \frac{x+9}{x-9} = \frac{28}{x^2-16x+63}$$

$$(x-7)(x-9)$$

$$D = \mathbb{R} \setminus \{7; 9\}$$

$$- \frac{7+x}{x-7} - \frac{x+9}{x-9} = \frac{28}{(x-7)(x-9)} \quad | \cdot (x-7)(x-9)$$

$$- (7+x)(x-9) - (x+9)(x-7) = 28$$

$$- (x^2 - 2x - 63) - (x^2 + 2x - 63) = 28$$

$$-x^2 + 2x + 63 - x^2 - 2x + 63 = 28 \quad | -126$$

$$-2x^2 = -98 \quad | \div (-2)$$

$$x^2 = 49 \quad | \pm \sqrt{}$$

$$x = \begin{cases} 7 \notin D ! \\ -7 \in D \checkmark \end{cases}$$

$$S = \{-7\}$$

$$8) \frac{2x}{x-3} - \frac{5}{x} = \frac{6x}{3x-9} + \frac{2}{3x}$$

$$3(x-3)$$

$$D = \mathbb{R} \setminus \{0; 3\}$$

$$\frac{2x \cdot 3x}{(x-3)3x} - \frac{5 \cdot 3(x-3)}{x \cdot 3(x-3)} = \frac{6x \cdot x}{3(x-3) \cdot x} + \frac{2(x-3)}{3x \cdot (x-3)} \quad | \cdot 3x(x-3)$$

$$6x^2 - 15(x-3) = 6x^2 + 2(x-3) \quad | -6x^2$$

$$-15x + 45 = 2x - 6 \quad | +15x+6$$

$$51 = 17x \quad | :17$$

$$3 = x \notin D !$$

$$S = \emptyset$$

$$9) \frac{x-1}{x^2+3x} + \frac{2}{x} + \frac{9}{2x+6} = 0$$

$$x(x+3) \quad 2(x+3)$$

$$D = \mathbb{R} \setminus \{0; -3\}$$

$$\frac{(x-1) \cdot 2}{x(x+3) \cdot 2} + \frac{2 \cdot 2(x+3)}{x \cdot 2(x+3)} + \frac{9 \cdot x}{2(x+3) \cdot x} = 0 \quad | \cdot 2x(x+3)$$

$$2(x-1) + 4(x+3) + 9x = 0$$

$$2x-2 + 4x+12 + 9x = 0$$

$$15x + 10 = 0$$

$$15x = -10$$

$$x = -\frac{10}{15} = -\frac{2}{3} \in D \checkmark$$

$$S = \left\{ -\frac{2}{3} \right\}$$

$$10) \frac{1}{x+6} - \frac{1}{x-6} - \frac{2x-10}{x^2-36} = 0$$

$$(x-6)(x+6)$$

$$D = \mathbb{R} \setminus \{6; -6\}$$

$$1 \cdot (x-6) - 1 \cdot (x+6) - (2x-10) = 0$$

$$x-6 - x-6 - 2x+10 = 0$$

$$-2x-2 = 0$$

$$-2 = 2x$$

$$x = -1 \in D \checkmark$$

$$S = \{-1\}$$

$$11) \frac{x+2}{x^2-8x+15} - \frac{1}{x-3} + \frac{x}{x-5} = 1$$

$$(x-3)(x-5)$$

$$D = \mathbb{R} \setminus \{3; 5\}$$

$$x+2 - 1 \cdot (x-5) + x(x-3) = 1 \cdot (x-3)(x-5)$$

$$x+2 - x+5 + x^2-3x = x^2-8x+15 \quad | -x^2$$

$$-3x+7 = -8x+15$$

$$5x = 8$$

$$x = \frac{8}{5} \in D \checkmark$$

$$S = \left\{ \frac{8}{5} \right\}$$

$$12) \left(1 - \frac{1}{x}\right)^2 = 9$$

$$D = \mathbb{R} \setminus \{0\}$$

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$$1 - \frac{2}{x} + \frac{1}{x^2} = 9$$

$$\frac{1x^2}{x^2} - \frac{2 \cdot x}{x \cdot x} + \frac{1}{x^2} = \frac{9x^2}{x^2} \quad | \cdot x^2$$

$$x^2 - 2x + 1 = 9x^2$$

$$-8x^2 - 2x + 1 = 0$$

$$\Delta = 4 - 4 \cdot (-8) \cdot 1 = 4 + 32 = 36$$

$$x_{1,2} = \frac{2 \pm 6}{-16} \rightarrow \frac{8}{-16} = -\frac{1}{2} \in D \checkmark$$

$$\frac{-4}{-16} = \frac{1}{4} \in D \checkmark$$

$$S = \left\{ \frac{1}{4}; -\frac{1}{2} \right\}$$

$$13) \frac{3x-1}{x-2} + \frac{2x-1}{x+1} = \frac{3-6x}{(x-2)(x+1)}$$

$$D = \mathbb{R} \setminus \{2; -1\}$$

$$(3x-1)(x+1) + (2x-1)(x-2) = 3-6x$$

$$3x^2 + 2x - 1 + 2x^2 - 5x + 2 = 3 - 6x$$

$$5x^2 - 3x + 1 = 3 - 6x \quad | -3 + 6x$$

$$5x^2 + 3x - 2 = 0$$

$$\Delta = 9 - 4 \cdot 5 \cdot (-2) = 49$$

$$x_{1,2} = \frac{-3 \pm 7}{10} \begin{cases} \frac{-10}{10} = -1 \notin D! \\ \frac{4}{10} = \frac{2}{5} \in D \checkmark \end{cases}$$

$$S = \left\{ \frac{2}{5} \right\}$$

$$14) \frac{3x}{x-4} - 2 = \frac{12}{x^2-16} \quad | \cdot (x+4)(x-4)$$

$$D = \mathbb{R} \setminus \{-4; 4\}$$

$$3x(x+4) - 2(x+4)(x-4) = 12$$

$$3x^2 + 12x - 2(x^2 - 16) = 12$$

$$3x^2 + 12x - 2x^2 + 32 = 12 \quad | -12$$

$$x^2 + 12x + 20 = 0$$

$$(x+10)(x+2) = 0$$

$$x_1 = -10 \in D \checkmark$$

$$x_2 = -2 \in D \checkmark$$

$$S = \{-10; -2\}$$

$$15) \frac{x+2}{x-2} + \frac{x-2}{x+2} = -\frac{10}{3}$$

$$D = \mathbb{R} \setminus \{-2; 2\}$$

$$\frac{(x+2)(x+2)3}{(x-2)(x+2)3} + \frac{(x-2)(x-2)3}{(x+2)(x-2)3} = -\frac{10(x+2)(x-2)}{3(x+2)(x-2)} \quad | \cdot 3 \cdot (x-2) \cdot (x+2)$$

$$3(x+2)^2 + 3(x-2)^2 = -10(x+2)(x-2)$$

$$3(x^2 + 4x + 4) + 3(x^2 - 4x + 4) = -10(x^2 - 4)$$

$$3x^2 + 12x + 12 + 3x^2 - 12x + 12 = -10x^2 + 40$$

$$6x^2 + 24 = -10x^2 + 40 \quad | +10x^2 - 40$$

$$16x^2 - 16 = 0$$

$$x^2 - 1 = 0$$

$$(x+1)(x-1) = 0$$

$$x_1 = 1 \in D \checkmark$$

$$x_2 = -1 \in D \checkmark$$

$$S = \{-1; 1\}$$

$$16) \frac{4-x}{2x-1} - \frac{1}{x+3} = \frac{7}{2x^2+5x-3} \quad | \cdot (2x-1)(x+3)$$

$$D = \mathbb{R} \setminus \{-3; \frac{1}{2}\}$$

$$(4-x)(x+3) - 1 \cdot (2x-1) = 7$$

$$4x - 3x - x^2 + 12 - 2x + 1 = 7$$

$$-x^2 - x + 13 = 7$$

$$x^2 + x - 6 = 0$$

$$(x+3)(x-2) = 0$$

$$x_1 = -3 \notin D!$$

$$x_2 = 2 \in D \checkmark$$

$$S = \{2\}$$